Roll No. ....

41181

B. Sc. (Pass Course) 4th Semester Examination – May, 2019

MATHEMATICS - I (SEQUENCES AND SERIES)

Paper : 12BSM241

Time : Three hours] ***[Maximum Marks : 40***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting at least ***one*** question from each Section. Section V is ***compulsory***.

SECTION – I

1. (a) Prove that set of rationals is not order complete. 7

(b) If x and y are two positive real numbers, then there exists a natural number n such that $nx > y$. 7

2. (a) Prove that every set satisfying Heine Borel property is a compact set. 7

(b) The derived set of any set is a closed set. Prove it.

P. T. O.

SECTION – II

3. (a) Let $< a_n >$ be a sequence s. t. $a_n \neq 0$ far all $n \in N$ and $\frac{a_n + 1}{a_n} \to l$ If $|l| < 1$ then $\underset{n \to \infty}{lt} a_n = 0$.

(b) Show that :

$$\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots\ldots + \frac{1}{(2n)^2} \right] = 0$$

4. (a) Test the convergence of the series $\frac{1}{1+x} + \frac{x}{1+x^2} + \frac{x^2}{1+x^3} + \ldots\ldots\ldots$ where $x > 0$.

(b) If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\underset{n \to \infty}{lt} a_n = 0$. Is the converse true ? If not show by an example ?

SECTION – III

5. (a) If $\sum_{n=1}^{\infty} a_n = \frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \ldots\ldots\ldots$ then show that cauchy root test establishes the convergence of the series. $\sum_{n=1}^{\infty} a_n$ while's Alembert's ratio test fails to do so.

(b) Test convergence of series :

$$1+\frac{x}{2}+\frac{21}{3^2}x^2+\frac{31}{4^3}x^3+\frac{41}{5^4}x^4 \ldots\ldots (x>0).$$

6. (a) Discuss convergence of series $\sum_{n=1}^{\infty}\frac{(n!)^2}{2n!}$

where $x>0$.

(b) Using Cauchy's condensation test, discu

convergence of $\sum_{n=2}^{\infty}\frac{\log n}{n}$

SECTION – IV

7. (a) Prove that if $\sum_{n=1}^{\infty} a_n$ is conditionally converge

then the series of its positive and the series of negative terms are both divergent.

(b) Test the convergence of the series :

$$\sum_{n=3}^{\infty}\frac{(n^3+1)^{1/3}-n}{\log n}$$

8. (a) Discuss the convergence of infinite product :

$$\prod_{n=1}^{\infty}\left(1+\frac{x}{n}\right),\ x\neq 0$$

(b) Show that the series $\left(1-\frac{2}{3}\right)+\left(1-\frac{8}{9}\right)+\left(1-\frac{26}{27}\right)+$ .... is convergent but when parenthesis are removed, it oscillates finitely.

SECTION –V

9. (a) Give examples to show that supremum of a set may or may not belong to the set. 2

(b) Give example of a set which has three limit points. 2

(c) Define absolute convergence and conditional convergence of $\sum_{n=1}^{\infty} a_n$. 2

(d) Give an example of a sequence which is bounded but not monotonic. 2

(e) Show that the series : 2

$$1^2+2^2+3^2+\ldots\ldots \text{ diverges to } +\infty$$

(f) Show that infinite product $\prod_{n=1}^{\infty}\left(1+\frac{1}{n^2}\right)$ is convergent. 2